

Prerequisite course for actuary exam

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Relation and function

Cartesian plane

What is a Cartesian plane?

What is a relationship?

Plot the following relation on a Cartesian plane.

$$x^2 + y^2 = 16$$

$$X + Y = 4$$

What is a function?

Determine if the following are function:

$$y = e^x$$

$$1 = \frac{x^2}{y}$$

$$x^2 + y^2 = 4$$

$$x + y^2 = 4$$

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Function example

$$X^2 + Y^2 = a^2$$

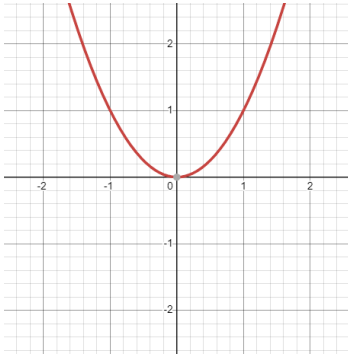
$$Y = X^2$$

$$y = e^x$$

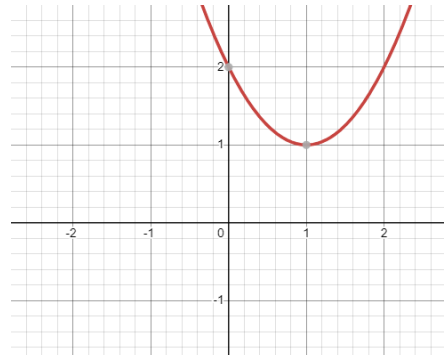
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Function shifting

$$F(x)=y \rightarrow F(x-a)=y-b$$



$$F(x) = x^2$$



$$f(x) - 1 = (X - 1)^2$$

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Series

Arithmetic series

$$a + (a + d) + (a + 2d) + \cdots + (a + (n - 1)d) = na + d \cdot \frac{n(n - 1)}{2}$$

Proof:

Sample questions

1. $1 + 2 + 3 + 4 + \cdots + 10 =$
2. $3 + 8 + 13 + 18 + \cdots + 68 =$
3. $2 + 5 + 8 + 11 + 14 + \cdots + 35 =$

Geometric series

For finite terms

$$a + ar + ar^2 + \dots + ar^{n-1} = a \times \frac{1 - r^n}{1 - r}$$

For infinite terms

$$a + ar + ar^2 + \dots = \frac{a}{1 - r} \text{ when } |r| < 1$$

Proof:

Sample questions

1. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \dots$

2. $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

3. $5 + 15 + 45 + \dots + 1215$

Taylor series

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

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Calculus

Limit

Notation

$$\lim_{x \rightarrow a} f(x)$$

Simple example

1. $\lim_{x \rightarrow 2} x - 2$

2. $\lim_{x \rightarrow 2} \frac{(x-2)(x-3)}{x-2}$

3. $\lim_{x \rightarrow 0} \frac{x^2 - 3x}{x}$

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When doesn't limit exist?

- When the right and left sides of a function approach different values.

$$\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x)$$

Example:

1. $\lim_{x \rightarrow 0} \frac{|x|}{x}$

2. $\lim_{x \rightarrow 0} \frac{1}{x^2}$

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3. $\lim_{x \rightarrow 0} \frac{1}{x}$

4. $\lim_{x \rightarrow 2} \frac{1}{2-x}$

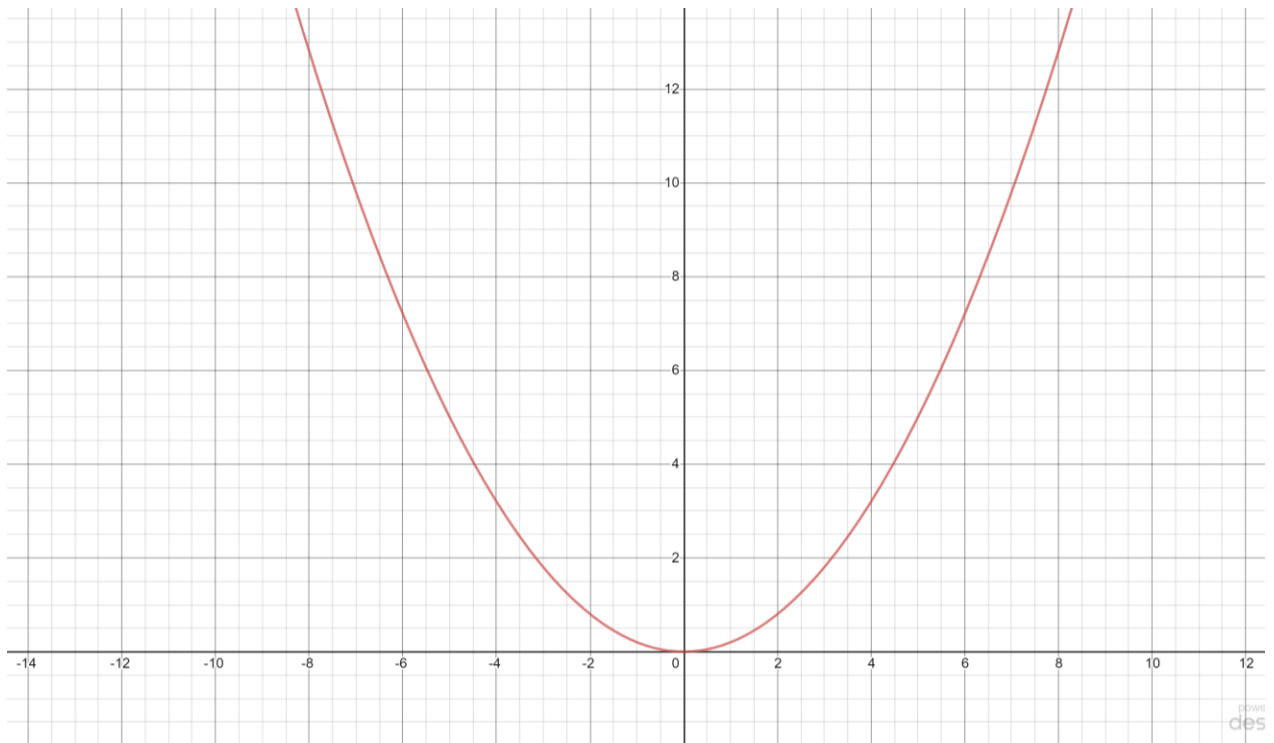
5. $\lim_{x \rightarrow 2} \frac{1}{\sqrt{x}}$

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Derivative

The derivative of $f(x)$ at $x = x_0$ is denoted $f'(x)$ or $\frac{df}{dx}$

$f'(x)$ is the slope of the graph at point x .

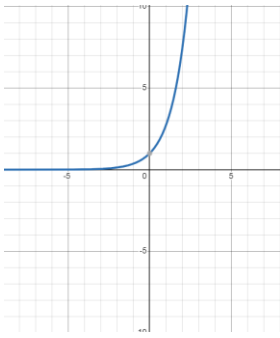


Definition of differentiation:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

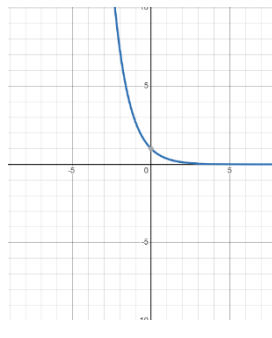
Derivative

first order - $f'(x)$



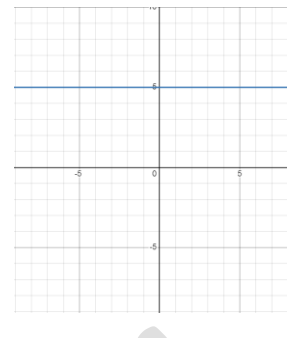
$$y = e^x$$

$$f'(x) > 0$$



$$y = e^{-x}$$

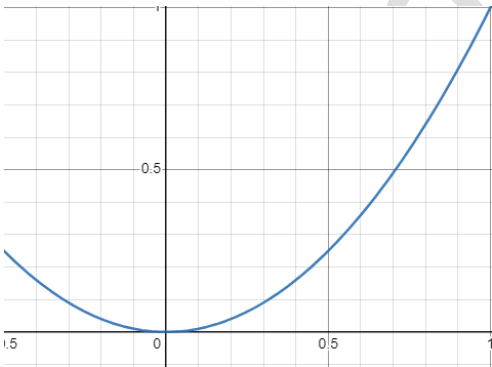
$$f'(x) < 0$$



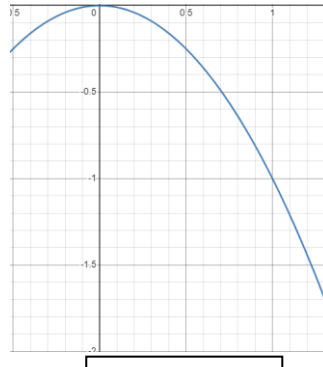
$$y = 5$$

$$f'(0) = 0$$

Second order - $f''(x)$



$$f''(x) > 0$$



$$f''(x) < 0$$

continuous and differentiable

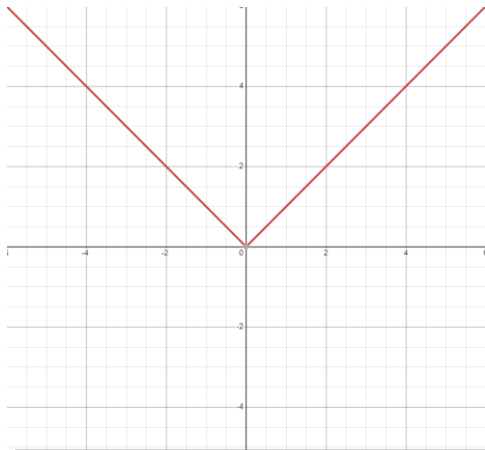
Condition for continuous at point c :

1. $f(c)$ exist
2. $\lim_{x \rightarrow c} f(x) = f(c)$

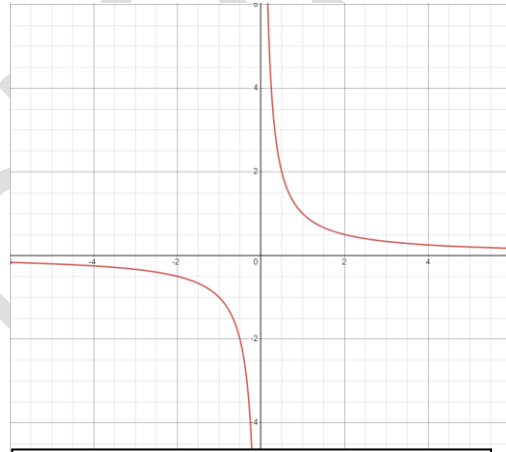
Condition for differentiable at point c :

1. $f'(c)$ exist

Example

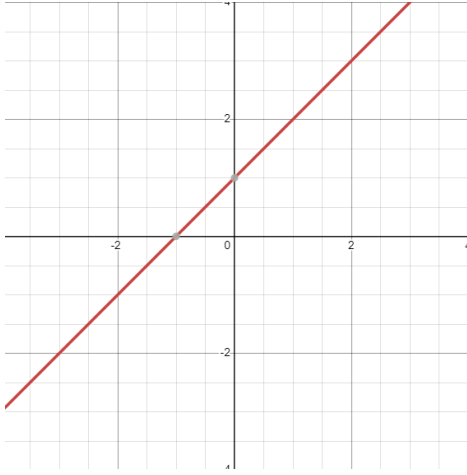


$$y = |x|$$

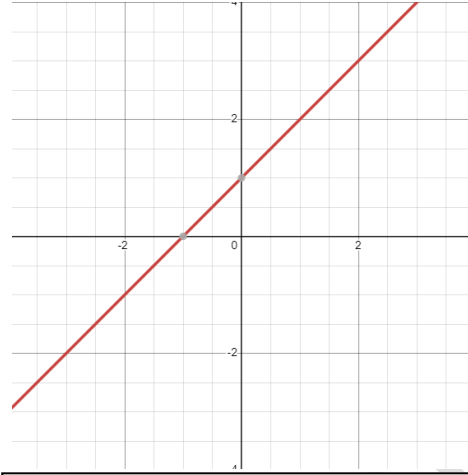


$$y = \frac{1}{x}$$

Derivative



$$y = \frac{x^2 - 1}{x - 1}$$



$$y = x + 1$$

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The Power Rule

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

And

$$\frac{d(C)}{dx} = 0$$

When C is a constant

Sample questions

1. $\frac{d}{dx} x^3 =$

2. $\frac{d}{dx} x^{-4} =$

3. $\frac{d}{dx} 8 =$

4. $\frac{d}{dx} x =$

5. $\frac{d}{dx} \sqrt{x} =$

General Derivative rules

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$$

$$\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))$$

Sample questions

1. $\frac{d}{dx} 5x^3 =$

2. $\frac{d}{dx}(12x^4 + 4x^{\frac{3}{2}}) =$

3. $\frac{d}{dx} 8(x^{-1} + x^{-2}) =$

4. $\frac{d}{dx} 3\sqrt{x} =$

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$$

Sample questions

1. $\frac{d}{dx}(x^3)(x^4) =$

2. $\frac{d}{dx}(x^2 + 1)(x^4 + 2) =$

3. $\frac{d}{dx}(x^2 + x)(x^2 + 1) =$

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Quotient Rule

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

Sample questions

1. $\frac{d}{dx} \frac{x^4}{x^3}$

2. $\frac{d}{dx} \frac{(x^2+1)}{(x^3+2)}$

3. $\frac{d}{dx} \frac{(x^2+x)}{(x^2+1)} =$

Other Derivative

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

Notes

$$\ln(x) = (\log_e x)$$

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Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Sample Questions

1. $\frac{d}{dx}e^{2x} =$

2. $\frac{d}{dx}\ln(x^3) =$

3. $\frac{d}{dx}(x^2 + x + 2)^2 =$

4. $\frac{d}{dx}e^{x^2} =$

5. $\frac{d}{dx}\ln(e^x + 1) =$

6. $\frac{d}{dx}e^{-5x} =$

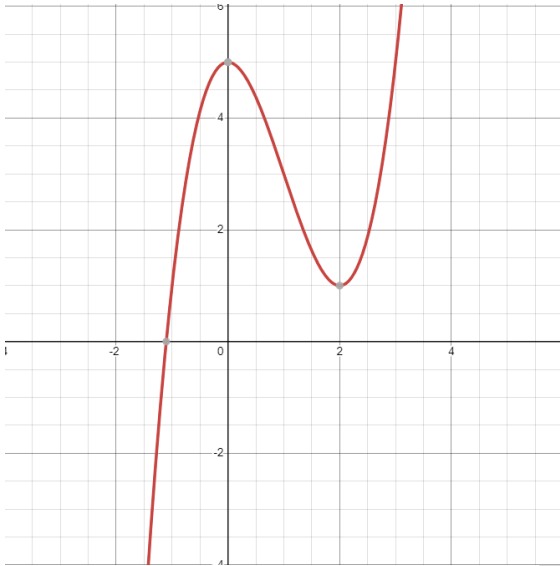
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Optimization

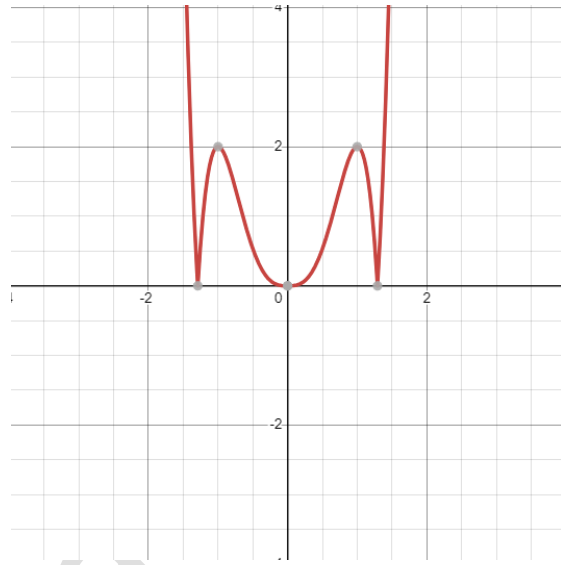
Find the maximum and minimum value of a function.

$$f'(x) = 0 \text{ or not define, solve for } X$$

Example



$$y = x^3 - 3x^2 + 5$$



$$y = |x^3| - 3x^2 + 5$$

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Indefinite integration

$\int f(x)dx$ mean the antiderivative.

$$F'(x) = f(x) \text{ if and only of } \int f(x)dx = F(x) + c$$

Power rule

$$\int x^r dx = \frac{x^{r+1}}{r+1} + c$$

Sample Questions

1. $\int x^9 dx =$

2. $\int 3x^2 dx =$

3. $\int x^5 dx =$

4. $\int \frac{1}{x^3} dx =$

5. $\int \sqrt{x} dx =$

6. $\int \frac{1}{\sqrt[3]{x}} dx =$

Other integration rule

$$\int (ag(x) \pm bh(x))dx = a \int g(x)dx \pm b \int h(x) dx + c$$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Sample Questions

1. $\int x^2 + 3x dx =$

2. $\int x^4 + 4 dx =$

3. $\int \sqrt{x} + 2x dx =$

4. $\int \frac{x^3+1}{x} dx =$

5. $\int (x + 1)(x - 1) dx =$

6. $\int 3e^x - 2 dx =$

U-Substitution

U-substitute is the inverse of chain rule.

1. Choose a new variable U
2. Compute $\frac{du}{dx}$
3. Replace all x's and dx with u and du
4. Evaluate the integration with variable U
5. Substitute back x in all U

Sample questions

1. $\int (x^3 + 1)^2 x^2 dx$

2. $\int x^2 e^{\frac{x^3}{4}} dx$

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3. $\int x\sqrt{2-x} dx$

4. $\int \frac{x^2}{x^3+5} dx$

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Integration by parts

Comes from product rule

$$1. \frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

$$2. f(x) \cdot g'(x) = \frac{d}{dx}[f(x) \cdot g(x)] - f'(x) \cdot g(x)$$

$$3. \int f(x) \cdot g'(x) dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x) dx$$

Or in shorter terms " $\int u dv = uv - \int v du$ "

Sample questions

$$1. \int x e^{-ax} dx$$

$$2. \int x \ln(x) dx$$

$$3. \int \ln(x) dx$$

4. $\int \frac{\ln(x)}{x^2} dx$

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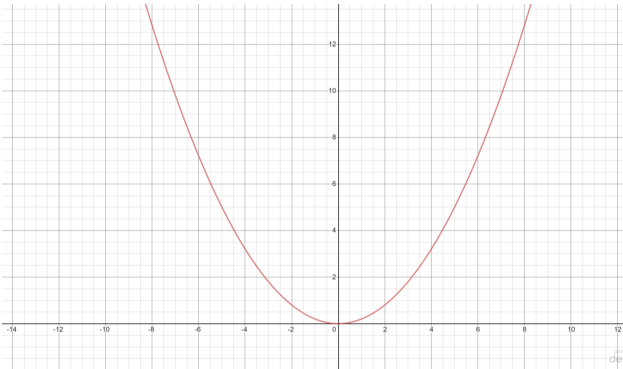
Definite integration

$$\int_a^b f(x) dx$$

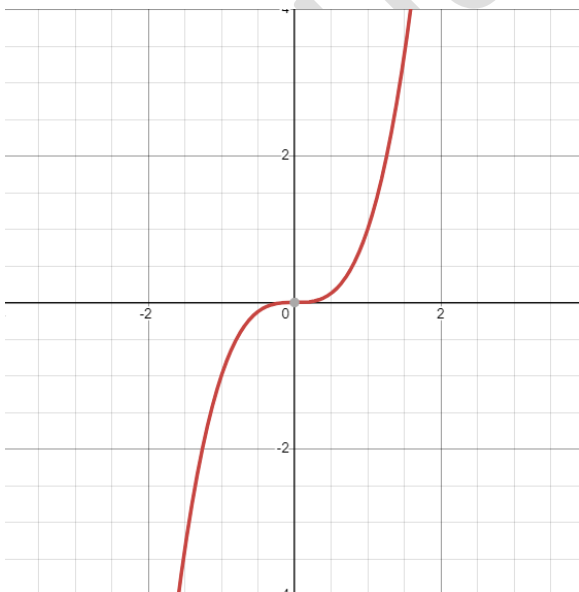
Is the area below the graph from a to b.

Example:

1. $F(X) = x^2$



2. $F(X) = x^3$



Fundamental theorem of calculus, Part 1:

If

$$F(x) = \int f(x)dx$$

Then

$$\int_A^B f(x)dx = F(A) - F(B)$$

Example

1. Find the area below the graph of

$$f(x) = x^3$$

From

$$-1 < x < 3$$

2. Find $\int_{-1}^3 f(x)dx$

Fundamental theorem of calculus, Part 2:

If

$$F(x) = \int_a^x f(y)dy$$

Then

$$F'(x) = f(x)$$

Example:

1. $f(x) = x^2$

2. $f(x) = x^3$

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